Controller design for two-wheeled self-balancing vehicles using feedback linearisation technique

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Abstract: Two-wheeled self-balancing vehicles have been extensively used because of their unique capabilities such as reducing the traffic problems and being environment friendly. The main challenge in these vehicles is to design robust controllers capable of creating smooth and safe movement. This paper focuses on design of such a controller using the feedback linearisation technique. The modelled system comprises of a single-link inverted pendulum, which simulates the rider's body, mounted atop a two-wheeled platform. While the base is being externally disturbed, the designed controllers guarantee the stability of the system and keep the rider along its equilibrium situation. In order to validate the effectiveness of proposed control schemes, some sets of simulation studies are carried out on different smooth and non-smooth surfaces. It is finally shown that besides having stability on smooth surfaces, the system behaves stably on non-smooth trajectories.

Keywords: two-wheeled vehicle; self-balancing vehicle; dynamic equations; feedback linearisation; non-smooth trajectory.

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1 Introduction

Two-wheeled mobile vehicle is a machine whose movement is based on two separately driven wheels placed on either side of the robot body (Maddahi et al., 2012a). Using differential drive mechanism, the vehicle is capable of changing its direction by varying the relative angular velocity of wheels, and therefore does not require an additional steering motion. During the vehicle motion, the centre of rotation may fall anywhere in the line joining the wheels together (Maddahi et al., 2012b). This uncertainty usually creates some inaccuracies, which is more challenging when the vehicle carries an inertial and massy body such as human. A solution, to reduce the occurred inaccuracy is to design a stable controller which is capable of keeping the vehicle and rider along desired situations.

Control of self-balancing systems has been ever of concern amongst the engineers. Two-wheeled vehicles, as one of the self-balancing systems, have attracted more attentions of researchers in recent decade. The idea of controlling these systems arises from the control problems of inverted pendulum (IP) which were of concern by the middle of recent decade (Bradshaw and Shao, 1996; Eker and Astrom, 1996; Mills et al., 2009). Saifizul et al. (2006) developed an intelligent controller for self-erecting IP via adaptive neuro-fuzzy interface system, and designed a position-velocity controller to swing up the pendulum considering physical behaviour. Sukontanakarn and Parnichkun (2009) proposed a real-time optimal control algorithm for keeping a rotary IP self-balancing and used a linear quadratic regulator (LQR) controller to balance the pendulum. Also, Zhong and Rock (2001) used a LQR to optimise the control gains utilised in the feedback controller. Moreover, Yamakita et al. (1992) proposed a variable structure system (VSS) adaptive control method for the IP of rotation type while assumed that the pendulum is simple rod for the stabilisation of the rotating arm. Slotine and Li (1991) developed an adaptive sliding mode controller satisfying the desirable properties of nonlinear systems with uncertainty, and proved the validity of the method which was used to the robot manipulator control. Furthermore, some studies have been done on self-balancing two-wheeled robots (SBTWR). To name a token, Yau et al. (2009) proposed a robust control method applied to a self-balancing SBTWR. They investigated both dynamic analysis and the control of this type of robot which is inherently unstable. It was followed by proposing an appropriate sliding surface to ensure the stability of the controlled close-loop system in sliding mode by Lyapunov stability theory. Also, Grasser et al. (2002) manufactured a small two-wheeled robot named JOE. This robot was difficult to control due to existence of friction force and light weight of corresponding mounted pendulum. One of the impressive studies, done in this area, is research performed by Lee and Jung (2009) on a mobile IP robot system. They proposed a sensor fusion technique of low cost sensors, i.e., gyro- and tilt sensory systems, to measure the balancing angle of the IP robot system using Kalman filter based on filtered sensor data.

Apart from any research carried out on IPs and SBTWRs, an extensive attention has been paid toward the control problem of two-wheeled self-balancing vehicles. Since, the safety of the human riding on the SBTWRs is very significant and applied control systems must guarantee the stability and the safety of the system in different situations, the vehicle must be controlled more accurate than the mentioned systems. Although the linear control of such systems results in good practical responses, the domain of attraction is extremely small (Aracil et al., 2008). Therefore, the need of controlling the nonlinear model of these vehicles is one of the interests of scholars and engineers in this area. The more accurate the control method is, the safer the vehicle will be. Madero et al. (2010) developed a nonlinear control law for two-wheeled self-balanced vehicles based on forwarding and proposed a Lyapunov function that allows obtaining an estimation of the domain of attraction for the resultant law. Furthermore, Tsai et al. (2010) used radial-basis-function neural networks (RBFNNs) for a two-wheeled scooter to achieve self-balancing and yaw control.

Since the equilibrium point of two-wheeled vehicles is inherently unstable, therefore, the accurate control of such systems is very significant in order to guarantee the safety of the rider. There exist different techniques to be used for designing the stable controller for such systems. Amongst them, the feedback linearisation method is simpler to implement. On the other hand, sliding mode control (SMC) is a useful method for tracking problems in robotic systems, and the problem of stabilising two-wheeled vehicles is not a tracking problem. The authors' experiences on the use of the SMC method, to stabilise two-wheeled vehicles, did not result in reasonable practical features due to the existence of sign function in high frequency control signal. In other words, FL method leads to better practical responses as compared to the SMC method. In the SMC method, the high oscillating chattering due to the discontinuity surface in terms of control signal can spoil the comfort of the rider while feedback linearisation method results in smoother response. Owing to the acceptable responses of this method, in many industrial applications, we intended to control this model of two-wheeled vehicles by feedback linearisation method.

In the current work, the control of the nonlinear model of two-wheeled self-balancing vehicle, capable of carrying human, is studied using feedback linearisation technique. The main contribution of this research is to control the vehicle while moving on a combined smooth and non-smooth surface. A set of simulation studies is done in which the vehicle is commanded to move on various surfaces. It is shown that the proposed control law is robust against different excitations exerted by the surface on the vehicle.

The rest of the paper is organised as follows. Section 2 presents the dynamic model of the vehicle. The control scheme is described in Section 3. The simulation results are shown in Section 4. The conclusion of the work is outlined in Section 5.

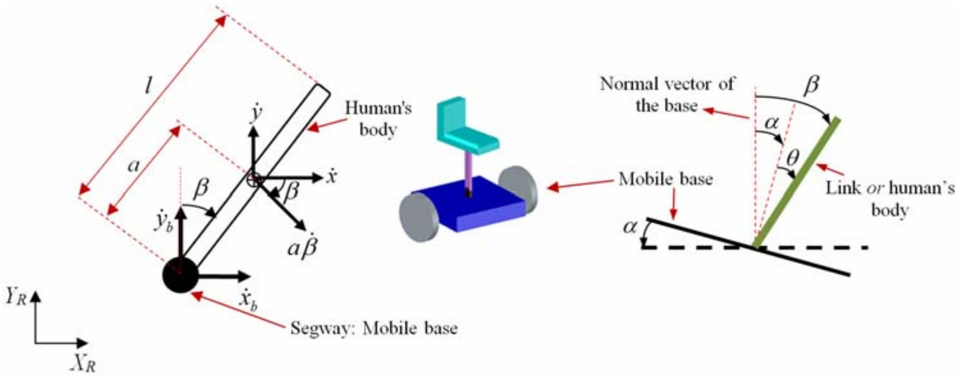
2 Dynamic model of two-wheeled vehicles

The considered dynamics model of the two-wheeled vehicle consists of two wheels, a differentially-driven system, a base and a 1-DOF link with a seat as the rider position shown in Figure 1. Note that the dynamic equations of the vehicle are obtained from two different views, as follows:

- *Case (a):* Vehicle moves: The aim of this section is to control the position of the rider. The control law is developed for the torque of motor driving the link.
- *Case (b):* Vehicle stops: In the second view, the vehicle is assumed to slow down until it fully stops. The application is in emergency situations like staying in traffic jam. Here, in addition to control the position of the rider, a control law should be developed for the torque of left wheel motor. Since the base of the two-wheeled vehicles is inherently unstable, thus, it must be maintained on its equilibrium point by exerting appropriate control signal as the torque on the relevant motor.

In Figure 1(a), $\beta = \theta + \alpha$, where θ is the angular displacement of the link with respect to the normal vector of base, and α represents the pitch angle which is the rotation of the base with respect to Z-axis [see Figure 1(b).

Figure 1 (a) Coordinate frames of two-wheeled self-balancing vehicle (b) Defining the angles (see online version for colours)



(a) (b)

2.1 Equations of motion in case (a)

There are two state variables β and $\dot{\beta}$ for case (a). Equation (1) describes the equation of motion of the vehicle which is obtained by implementing Lagrangian technique in two-dimensional movement. Obviously, only β is of concern as the generalised coordinate in Lagrange method.

$$\left(ma^2 + I\right)(\ddot{\beta}) + \left(ma\frac{R}{2}\right)(1+k)\ddot{\varphi}_L \cos(\beta) - ma(\ddot{y} - g)\sin(\beta) = T \tag{1}$$

where m is the mass of the rider's body (link). a and R represent the distance between the center of mass and the joint of link and the radius of right/left wheel, respectively. I denotes the link moment of inertia with respect to its center of mass. Moreover, T represents the torque of motor used to drive the link, and k represents the differential coefficient between right and left wheels. In addition, g is the gravitational acceleration, $\ddot{\beta}$ is referred to as the angular acceleration of link, and finally, $\ddot{y}$ symbolises the excitation which the surface exerts on the vehicle. In this representation, x is rephrased in terms of the angular displacement of left wheel, φ_L, as follows:

$$x = \frac{\left(x_L + x_R\right)}{2} = \frac{R}{2}(1+k)\varphi_L \tag{2}$$

In (2), $x_L = R.\varphi_L$ and $x_R = R.\varphi_R$ denote the linear displacement of left and right wheels, respectively. The angular displacement of left wheel, φ_L, is only considered in dynamic modelling, and φ_R (rotation of right wheel) is calculated by multiplying the φ_L to a constant coefficient, i.e., $\varphi_R = k\varphi_L$ which utilises a differentially-driven mechanism.

The state space model of the vehicle dynamics is obtained based on equation (1). The following states are defined for case (a):

$$\begin{cases} x_1 = \beta \\ x_2 = \dot{\beta} \end{cases} \tag{3}$$

Using equation (1), the following state space model is obtained:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{1}{A}\left[T - B\ddot{\varphi}_L + ma(\ddot{y} - g)\sin(x_1)\right] \end{cases} \tag{4}$$

where T denotes the control law which should be used to stabilise the system. A and B factors are defined as $A = I + ma^2$ and $B = (1+k)\left(m\frac{R}{2}a\right)\cos(x_1)$.

2.2 Equations of motion in case (b)

In case (b), the two-wheeled self-balancing vehicle reduces its speed and is compelled to stop in some situations such as traffic jam. The dynamic equations of this case are presented in equation (5). In equation (5), the angular displacement of link with respect to upright position (β), and the rotation of left wheel (φ_L) are considered as generalised variables in Lagrangian notation.

$$\begin{bmatrix} I + ma^2 & ma\frac{R}{2}(1+k)\cos(\beta) \\ ma\frac{R}{2}(1+k)\cos(\beta) & I_w(1+k) + \left(\frac{M_b}{4}(1+k)^2 + M_w\left(1+k^2\right)\right)R^2 + (1-k)J_\delta\left(\frac{R}{d}\right)^2 \end{bmatrix} \tag{5}$$
$$\begin{bmatrix} \ddot{\beta} \\ \ddot{\varphi}_L \end{bmatrix} + \begin{bmatrix} 0 \\ -ma\frac{R}{2}\sin(\beta)(\dot{\beta}) \end{bmatrix}(\dot{\theta} + \dot{\alpha}) + \begin{bmatrix} -ma\sin(\beta) \\ 0 \end{bmatrix}(\ddot{y}) + \begin{bmatrix} ma\sin(\beta) \\ 0 \end{bmatrix}(g) = \begin{bmatrix} T \\ T_{le} \end{bmatrix}$$

In equation (5), I_w is the left wheel moment of inertia with respect to its centre of mass. M_b and M_w represent the masses of base and wheel, respectively. Since, the yaw angle δ is considered in the modelling, therefore, J_δ is entered into the dynamic equations as the moment of inertia of the base with respect to Y-axis. d is referred to as the wheelbase, and T_{le} denotes the control torque of motor installed to drive the left wheel. Note that the yaw angle δ is defined as a linear function of x_L and x_R as below:

$$\delta = \frac{x_L - x_R}{d} = \frac{R}{d}(1-k)\varphi_L \tag{6}$$

It is easily understandable when $k = 1$, the yaw angle equals zero and it means that the vehicle does not deviate with respect to Y-axis.

Moreover, the effect of the friction force has not been considered in equation (5). By taking this force into account, the term T_{le} in equation (5) is replaced by the term $T_{le} - F_r.R$, in which, F_r is the rolling resistance force imposed by the road on the wheels and is presented as equation (7). The following equation is a valid empirical model for rolling resistance force used in many vehicle dynamic problems (Jazar, 2008). In equation (7), the effects of tire pressure (p), vehicle velocity (v_x) and normal force (F_z) on the rolling resistance force are considered.

$$F_r = \frac{K}{1{,}000}\left[5.1 + \frac{5.5\times 10^5 + 90F_Z}{p} + \frac{1{,}100 + 0.0388F_Z}{p}.v_x^2\right].F_z \tag{7}$$

The parameter K and p are set to 0.8 and 100 KPa for radial tires used in proposed two-wheeled vehicles. The normal force (F_z) is, also, calculated as follows:

$$F_z = \frac{1}{2}\left(M_b + 2M_w + m\right)g \tag{8}$$

According to equation (5), the following state variables are considered for case (b):

$$\begin{cases} x_1 = \beta & x_3 = \varphi_L \\ x_2 = \dot{\beta} & x_4 = \dot{\varphi}_L \end{cases} \tag{9}$$

And, the state space model of the system is expressed as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{B}{B^2 - AF}\left[T_{le} - Z - \dfrac{F}{B}(T - Q)\right] \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \dfrac{A}{AF - B^2}\left[T_{le} - Z - \dfrac{B}{A}(T - Q)\right] \end{cases} \tag{10}$$

where the abbreviations used in equation (10) symbolise the following terms:

$$\begin{cases} F = I_w(1+k) + \left(\dfrac{M_b}{4}(1+k)^2 + M_w\left(1+k^2\right)\right)R^2 + (1-k)J_\delta\left(\dfrac{R}{d}\right)^2 \\ Q = -ma\sin(x_1).\ddot{y} + mag\sin(x_1) \\ Z = -ma\dfrac{R}{2}\sin(x_1).x_2{}^2 \end{cases}$$

3 Derivation of control laws

The control laws are obtained by applying feedback linearisation technique on the derived equation(s) of motion in each case. Firstly, the output variables are identified followed by designing the control torques based on this technique (Slotine and Li, 1991). This section is also categorised into two categories as follows.

3.1 Design of control law for case (a)

In case (a), the output variable of the system is β, where:

$$\ddot{\beta} = \frac{1}{A}\left[T - B\ddot{\varphi}_L + ma(\ddot{y} - g)\sin(x_1)\right] \tag{11a}$$

Now, based on feedback linearisation method, the control parameter w is defined as follows:

$$w = \ddot{\beta} = \frac{1}{A}\left[T - B\ddot{\varphi}_L + ma(\ddot{y} - g)\sin(x_1)\right] \tag{11b}$$

By substituting equation (11b) in equation (4), we have:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = w \end{cases} \tag{12}$$

Equation (12) represents a linear form of state equations in terms of the control parameter w, and laws followed in classic control. Therefore, w is defined as a PD controller as follows:

$$w = \ddot{\beta}_d + k_d\dot{e} + k_p e \tag{13}$$

where k_p and k_d are the proportional and derivative coefficients of the PD controller. In the interim, $\ddot{\beta}_d$ denotes the desired angular acceleration of the link. $e = \beta_d - \beta$, and $\dot{e} = \dot{\beta}_d - \dot{\beta}$. β_d and $\dot{\beta}_d$ represent the desired output variable and the desired first derivative of output variable, respectively. By these definitions, e and $\dot{e}$ are physically referred to as the error signal and the derivative of error signal produced due to the differences between the actual and desired signals.

By combining equations (11b) and (13), the control law is obtained as follows:

$$T = A\left(\ddot{\beta}_d + k_d\dot{e} + k_p e\right) + B\ddot{\varphi}_L - ma(\ddot{y} - g)\sin(x_1) \tag{14}$$

The developed control law in equation (14) is the torque guarantees that the link (simulated as the rider) approaches its equilibrium point, and keeps the rider in upright position.

3.2 Design of control law for case (b)

Similar to the procedure implemented in case (a), to apply feedback linearisation technique on case (b), first, the output variables of the system must be obtained. With reference to equation (5), these output variables are β and φ_L; therefore, the control parameters w_1 and w_2 are defined as:

$$\begin{cases} w_1 = \ddot{\beta} = \dfrac{B}{B^2 - AF}\left[T_{le} - Z - \dfrac{F}{B}(T - Q)\right] \\ w_2 = \ddot{\varphi}_L = \dfrac{A}{AF - B^2}\left[T_{le} - Z - \dfrac{B}{A}(T - Q)\right] \end{cases} \tag{15}$$

Subsequently, equation (10) is re-arranged as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = w_1 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = w_2 \end{cases} \tag{16}$$

As shown, a linear form of state equations is again constituted in terms of the control parameters w_1 and w_2. Now, these parameters are defined as PD controllers same as what described for case (a):

$$\begin{cases} w_1 = \ddot{\beta}_d + k_{d,1}\dot{e}_1 + k_{p,1}e_1 \\ w_2 = \ddot{\varphi}_{L,d} + k_{d,2}\dot{e}_2 + k_{p,2}e_2 \end{cases} \tag{17}$$

In equation (17), $\ddot{\varphi}_{L,d}$ is the desired angular acceleration of the left wheel. $k_{d,1}$ and $k_{p,1}$ denote the derivative and proportional coefficients of control parameter w_1 while $k_{d,2}$ and $k_{p,2}$ are the corresponding coefficients of w_2. Meanwhile, we have:

$$\begin{cases} e_1 = \beta_d - \beta \\ \dot{e}_1 = \dot{\beta}_d - \dot{\beta} \\ e_2 = \varphi_{L,d} - \varphi_L \\ \dot{e}_2 = \dot{\varphi}_{L,d} - \dot{\varphi}_L \end{cases} \tag{18}$$

In this notation, $\varphi_{L,d}$ and $\dot{\varphi}_{L,d}$ represent the desired values for angular displacement and velocity of the left wheel, respectively. By substituting equation (17) into equation (15) and using Cramer method, the control torques are obtained as below:

$$\begin{cases} T = \dfrac{1}{\mu}\left[\left(\ddot{\beta}_d + k_{d,1}\dot{e}_1 + k_{p,1}e_1\right) - f_1 - \dfrac{g_1}{g_2}\left(\left(\ddot{\varphi}_{L,d} + k_{d,2}\dot{e}_2 + k_{p,2}e_2\right) - f_2\right)\right] \\ T_{le} = \dfrac{1}{\xi}\left[\left(\ddot{\beta}_d + k_{d,1}\dot{e}_1 + k_{p,1}e_1\right) - f_1 - \dfrac{A.F.g_1}{B^2.g_2}\left(\left(\ddot{\varphi}_{L,d} + k_{d,2}\dot{e}_2 + k_{p,2}e_2\right) - f_2\right)\right] \end{cases} \tag{19}$$

where A and B were defined in equation (4), and

$$\begin{cases} f_1 = \dfrac{B}{B^2 - AF}\left(-Z + \dfrac{FQ}{B}\right) & g_1 = \dfrac{B}{B^2 - AF} \\ f_2 = \dfrac{A}{AF - B^2}\left(-Z + \dfrac{BQ}{A}\right) & g_2 = \dfrac{A}{AF - B^2} \\ \mu = -\dfrac{F.g_1}{B} + \dfrac{B.g_1}{A} & \xi = g_1 - \dfrac{A.F.g_1}{B^2} \end{cases}$$

4 Simulation results

Simulation study was carried out in Matlab software using fourth order Runge-Kutta method as the integration routine in order to demonstrate the stability of the two-wheeled self-balancing model by applying feedback linearisation technique. The required vehicle and human specifications, used in simulations, are given in Table 1.

Table 1 System parameters used in simulation studies

Parameter	*Symbol*	*Value*
Moment of inertia of link with respect to its C.G.	I	3.1 kg.m^2
Mass of link	m	100 kg
Distance between the link C.G. and joint	a	0.6 m
Radius of the each wheel	R	0.15 m
Gearbox ratio between left wheel and right one	k	1
Gravitational acceleration	g	9.81 m/s^2
Mass of vehicle base	M_b	100 kg
Mass of each wheel	M_w	12 kg
Moment of inertia of the wheel with respect to its C.G	I_W	0.225 kg.m^2
Moment of inertia of the vehicle base with respect to Y-axis	J_{ch}	0.6 kg.m^2
Wheelbase	d	0.6 m
The angular acceleration of the left wheel which is effective in the dynamic of the vehicle in case (a)	$\ddot{\varphi}_L$	sin(t)

According to equation (1), the angular acceleration of the wheel $(\ddot{\varphi}_L)$ affects the link position. In order to evaluate how robust the controller is against the different dynamic conditions, the parameter $\ddot{\varphi}_L$ was considered equal to sin(t) in simulation of case (a). By this assumption, it was tried to consider a difficult condition for the dynamic of the vehicle. To show the applicability of the proposed control laws, the vehicle was moved on five different surfaces including smooth, pulse, ramp, unseen and sinusoid. Several trial and errors were done in order to tune the controller gains in each case. The final values of proportional and derivative coefficients are given in Table 2. The initial conditions were considered as below for each case:

$$\text{case (a):} \begin{cases} \theta(0) = 0.1 \\ \dot{\theta}(0) = 0 \end{cases} \tag{20}$$

$$\text{case (b):} \begin{cases} \theta(0) = 0.1 \\ \dot{\theta}(0) = 0 \\ \varphi_L(0) = 0.1 \\ \dot{\varphi}_L(0) = 0.02 \end{cases} \tag{21}$$

Table 2 Proportional and derivative control gains

Case (a)	k_p	28
	k_d	2
Case (b)	$k_{p,1}$	25
	$k_{d,1}$	3
	$k_{p,2}$	15
	$k_{d,2}$	2

4.1 Simulation results for case (a)

The results from the simulation of the vehicle movement on different surfaces were very similar to each other; therefore, the results of the simulation on *unseen* surface were *just* shown in order to avoid repetition. In this case, the parameter $\ddot{y}$ was chosen as a random function, i.e., it is assumed that the vertical acceleration $(\ddot{y})$, exerted on the vehicle due to surface irregularities, has random values during the simulation. Figure 2 depicts the random function showing the variations of angular displacement and velocity of link with respect to time are shown in Figure 3. As observed, both states reach the corresponding equilibrium point. This test confirms the theoretical analysis performed earlier. The control signal (torque of link) is shown in Figure 4. Looking at the output torque, as written in equation (14), it is confirmed that the control signal is smooth which is desirable.

Figure 2 Random function considered for parameter $\ddot{y}$

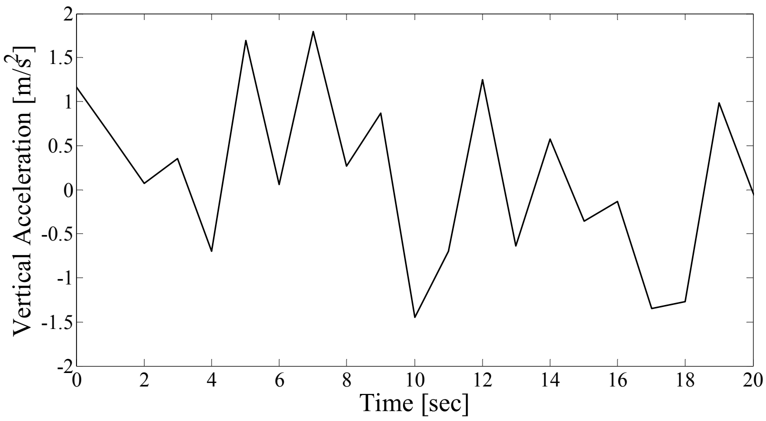
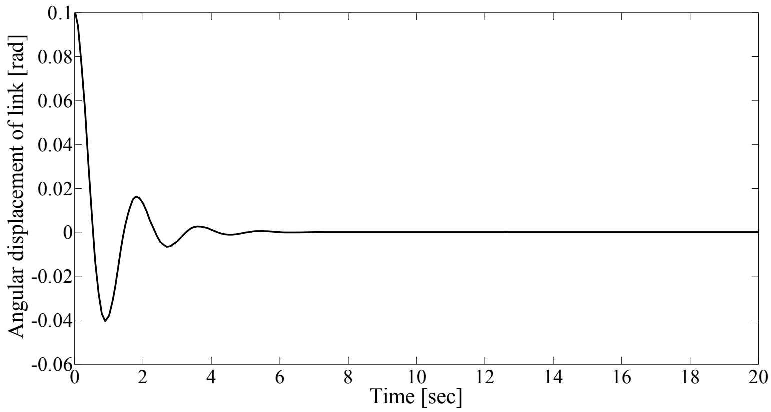
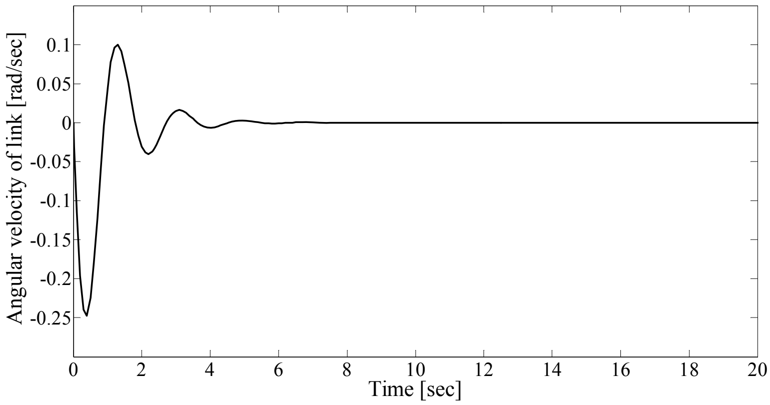


Figure 3 Variations of (a) angular displacement and (b) angular velocity of link with respect to time, case (a)



(a)

Figure 3 Variations of (a) angular displacement and (b) angular velocity of link with respect to time, case (a) (continued)



(b)

Figure 4 Control torque pertaining to the link, case (a)

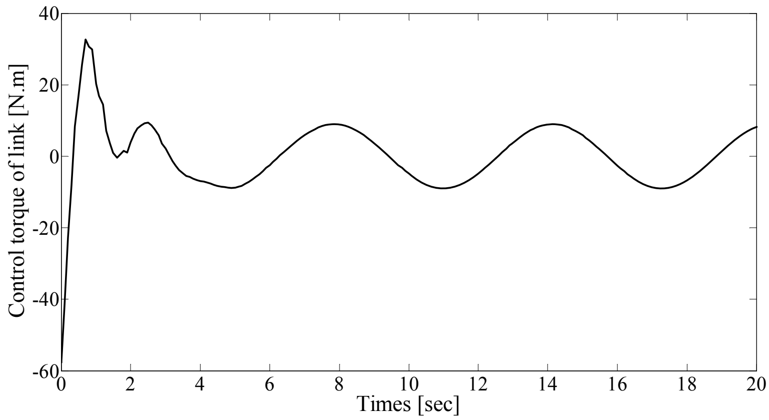


Figure 5 Effect of variations of left wheel angular acceleration on the link control torque

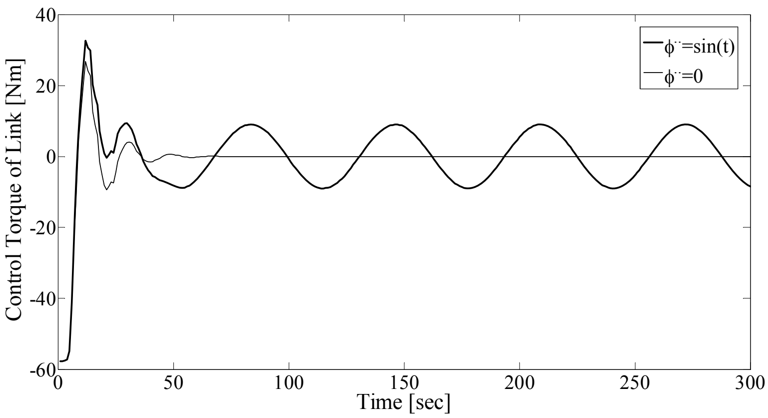
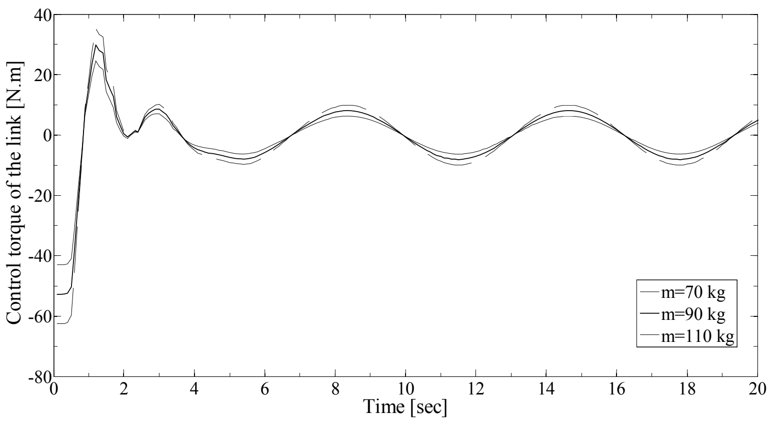


Figure 4 shows that the behaviour of torque is sinusoid-like function after 5 sec which is due to choosing $\ddot{\varphi}_L$ equal to sin(t). To show the effect of the angular acceleration of the on torque of link, the second set of simulations was carried out for case (a) in which the value of $\ddot{\varphi}_L$ was set to zero. The results showed that the torque of link approaches zero after around 50 sec. Figure 5 illustrates the effect of changing the $\ddot{\varphi}_L$ on the control torque of link, T.

In third set of simulation studies, the effect of the mass of rider on the control torque, T, was investigated. The simulation study was carried out on the model considering various values for m. Figure 6 illustrates how the behaviour of control torque can change by varying the mass of link varies from m = 70 kg to m = 110 kg. With reference to Figure 6, the more the link mass is, the more overshoot of control torque will be. Note that, in this investigation, the angular acceleration of the left wheel, $\ddot{\varphi}_L$, was set to sin(t).

Figure 6 Link control torque under variations of the link mass

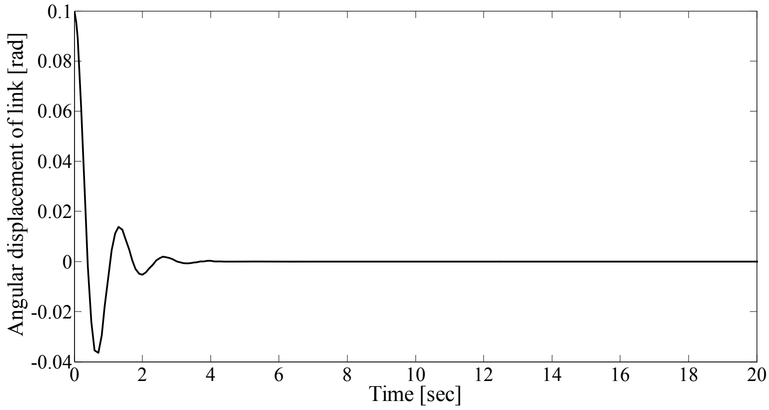


4.2 Simulation results for case (b)

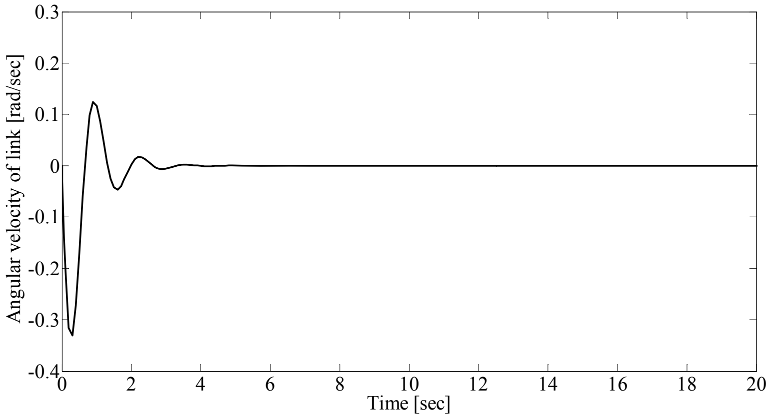
In the fourth set of simulations, it is assumed that the two-wheeled self-balancing vehicle slows down and then stops on a path whose surface imitates a random function. Figure 7 shows how the angular displacement and angular velocity of link varies with respect to time. The results of Figure 7 indicate that both states pertaining to link approach zero by applying the control signal T proposed in equation (19). The variations of angular displacement and velocity of left wheel are depicted in Figure 8. As observed both φ_L and its derivative with respect to time approach the equilibrium point (zero values) belonging to the base of the two-wheeled vehicle. This means that by exerting torque (T_{le}) presented in equation (19), the stability of the vehicle base is maintained while the vehicle slows down and finally stops.

As mentioned earlier, while the vehicle slows down, control signals, defined in equation (19), the stability of the combination of rider and vehicle is guaranteed. As illustrated in Figure 9, considering the initial conditions mentioned in equation (21), values of 50 N.m and 13 N.m (for T and T_{le} respectively) must be applied on motors in order to keep the link and the base along corresponding equilibrium points.

Figure 7 Variations of (a) angular displacement and (b) angular velocity of link, case (b)

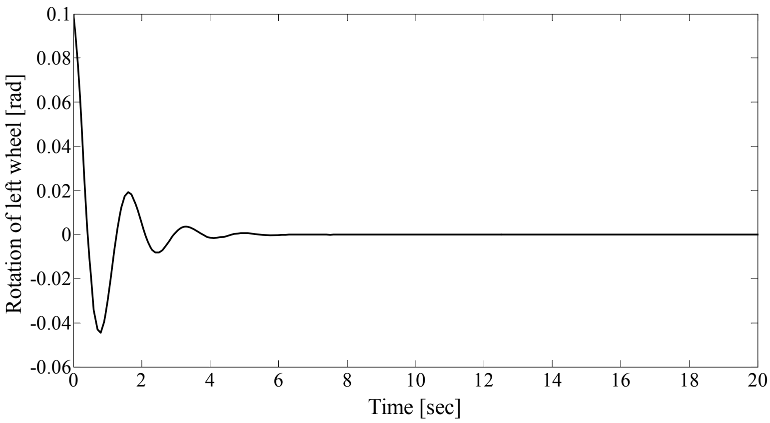


(a)



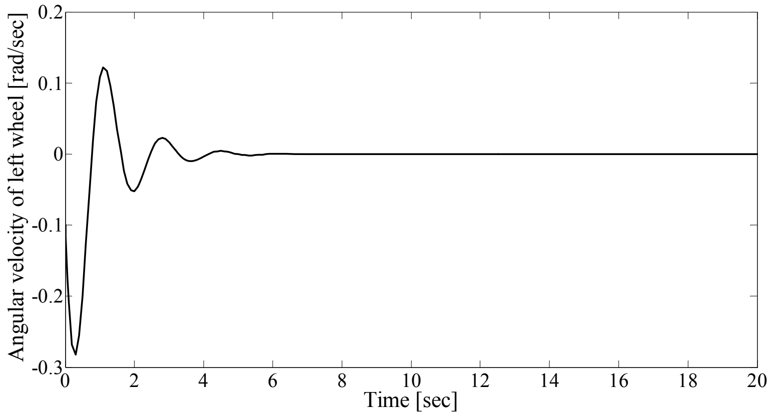
(b)

Figure 8 Variations of (a) angular displacement and (b) angular velocity of the left wheel, case (b)



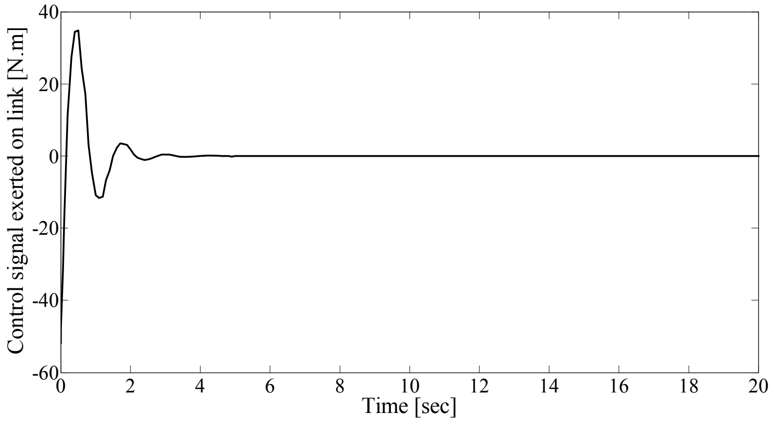
(a)

Figure 8 Variations of (a) angular displacement and (b) angular velocity of the left wheel, case (b) (continued)

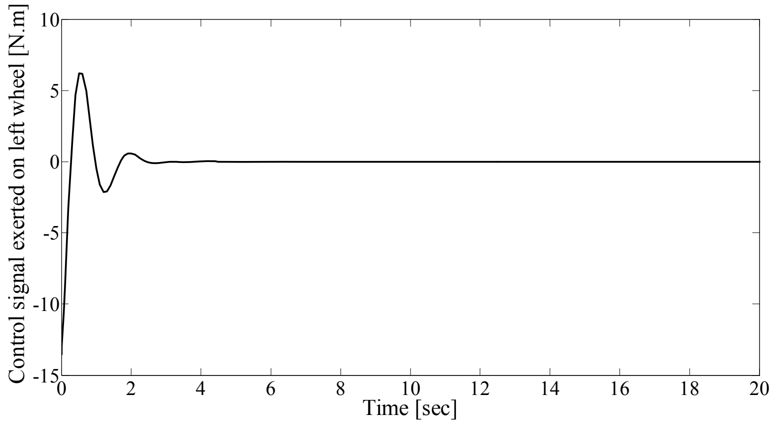


(b)

Figure 9 Control signal pertaining to (a) link and (b) left wheel, case (b)



(a)



(b)

Finally, the effect of friction on the control of the case (b) is investigated. With reference to equations (5) and (7), by imposing the friction force on the right hand side of equation (5), the behaviour of control torques varies with respect to condition in which the effect of friction is ignored. As observed in Figures 10 and 11, values of link control torque T_{le} increases up to 2 N.m in steady state by considering the effect of rolling resistance force on wheels while the vehicle tends to stop. Figure 11 illustrates the effect of friction, defined between wheel and road, on the control torque of the left wheel.

Figure 10 Effect of the friction between the wheel and road on the control torque of the link

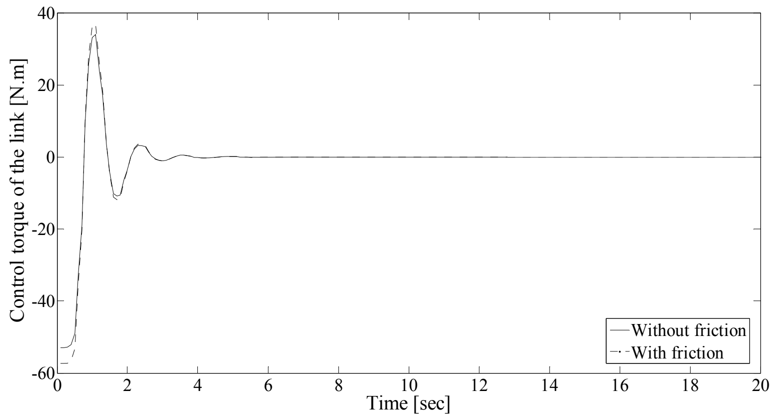
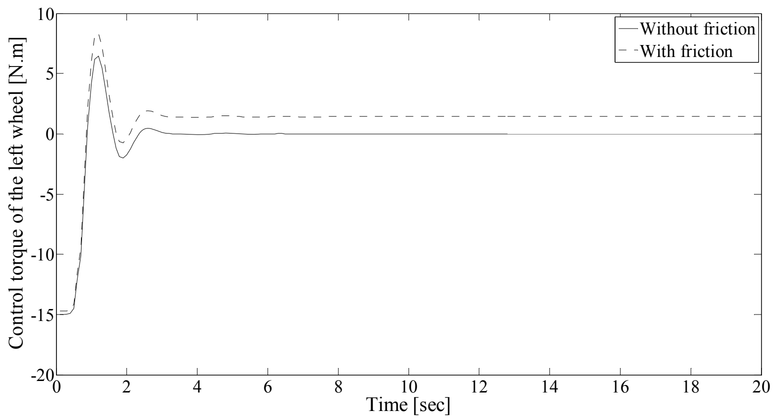


Figure 11 Effect of the friction between wheel and road on the control torque of the left wheel



5 Conclusions

The purpose of this paper was to design a stable control scheme for two-wheeled self-balancing vehicles. The focus was on systems consisting of a two-wheeled differentially-driven mobile vehicle, and a 1-DOF link, behaving representing the rider's body, which is mounted atop of the base. Firstly, the dynamic equations of the vehicle were derived from two different viewpoints: when the vehicle was in movement, and vehicle slowed down until it stopped. The appropriate controllers were designed using feedback linearisation technique. Afterward, the proposed controllers were validated by carrying out five sets of simulation studies on smooth and non-smooth surfaces. It was demonstrated that the simulated vehicle was stable, and the states converged to corresponding equilibrium values. Moreover, the simulation results show that the system is robust to rider's mass and existence of friction. Future work can focus on designing the stable controllers for this application, and validating the design process by performing the simulations using other stability techniques such as Lyapunov.

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